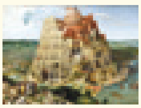
Logic Lectures

Gödel's Basic Logic Course at Notre Dame

Edited by
Miloš Adžić and Kosta Došen



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We are grateful to the Institute for Advanced Study in Princeton for granting us, through the office of its archivist Mr Casey Westerman, the permission to publish Gödel's unpublished notes; we were asked to give credit for that with the following text: "Works of Kurt Gödel used with permission of Institute for Advanced Study. Unpublished Copyright (1939) Institute for Advanced Study. All rights reserved."

Abbrev.⌊iated⌋ editorial introduction

Gödel taught a one-semester course in basic logic at the University of Notre Dame in the spring of 1939, when he turned 33. Among his unpublished writings in the Princeton University Library one can find notebooks with the manuscript of his notes for that course. The title *Logic Lectures*, which we gave to these notes, is suggested by the German "Log.⌊ik⌋ Vorl.⌊esungen⌋", or a variant of that, written on the front covers of the notebooks.

Besides the Notre Dame course Gödel taught a basic logic course in Vienna in the summer of 1935, notes for which, on 43 notebook pages (27 of which are numbered), made mainly of formulae and very little accompanying text in ordinary language, have been preserved in a manuscript at the same place. The notes for the Notre Dame course, which with their 427 notebook pages are ten times bigger, are more detailed and we think more important. Propositional logic is not much present in the Vienna notes.

We have published recently in [A. & D. 2016] a brief, and hence not complete, summary with comments of the Notre Dame notes, and an assessment of their importance. This preceding short paper is a natural introduction to this introduction, which is more oriented towards details concerning Gödel's text. We deal however here occasionally, in the paragraph on definite descriptions below and in the last few pages of this introduction, with some matters of logic and philosophy, partly in the sphere of the preceding paper, but not to be found there. Anyway, that paper enables us to abbreviate this introduction (which explains up to a point its title; the rest will be explained in a moment).

We will not repeat ourselves, and we will not give again all the references we gave in the preceding paper, but we want to mention however John Dawson, who in [Dawson] supplies biographical data on Gödel's stay at Notre Dame, John and Cheryl Dawson who in [Dawson 2005] set what we did with the Notre Dame notes as a task for Gödel scholars,[1] and Pierre Cassou-Noguès, who has published in [Cassou-Noguès 2009] a dozen printed pages extracted and edited from Gödel's manuscript of the Notre Dame course (this concerns pp. **1**.-**26**. of Notebook I, including small bits of Notebook 0, pp. **73**.1-**73**.7 of Notebook V, pp. **122**.-**125**., **134**.-**136**. of Notebook VI and pp. **137**.-**157**. of Notebook VII; altogether 60 notebook pages).[2]

[1]We are grateful to John Dawson for encouraging us to get into this publishing project.

[2]We have found sometimes useful Cassou-Noguès' reading of Gödel's manuscript, and we wish to acknowledge our debt. Our decipherment of the manuscript does not however

Besides the edited version of Gödel's text we have prepared another version of it, which we call the source version, and the present introduction should serve for both of them. This other, source, version is quite close to the original manuscript, and is meant to be a record of the additions and other interventions made in the manuscript to arrive at the edited version, and a justification of that later version.

Gödel used abbreviations in the manuscript of the notes quite a lot. For example, the second sentence and the beginning of the third of Notebook 0 of the manuscript are: "Accord. to this def the centr. part of log. must be the theory of inf and the theory of logically true prop. By a log true prop. I mean a prop. which is true for merely log reasons..." In the source version this is rendered as: "Accord.⌊ing⌋ to this def⌊inition⌋ the centr⌊al⌋ part of log.⌊ic⌋ must be the theory of inf⌊erence⌋ and the theory of logically true prop⌊ositions⌋. By a log⌊ically⌋ true prop.⌊osition⌋ I mean a prop.⌊osition⌋ which is true for merely log⌊ical⌋ reasons..." All the abbreviated words are typed in the source version as they occur in the manuscript, with a full stop after the abbreviation or without, together with their prolongation or decipherment within the parenthetical signs ⌊ and ⌋ to obtain the non-abbreviated, disabridged, word they are supposed to stand for, which one finds in the edited version. Sometimes whole words are omitted and they are restored in the source version within ⌊ and ⌋.

Using abbreviations may produce problems, which are however surmountable. For example, log., with or without full stop, stands for "logic", "logically" and "logical". Singular or plural has to be inferred from the context; "form.", with or without full stop, stands for "formula" or "formulas" (Gödel has the plural "formulas" while we here and in our comments use "formulae"; he says often "expression" for "formula"). Sometimes, but not very often, it is not obvious, and even not certain, what is the abbreviated word; for example, both "proposition" and "property" are abbreviated by "prop.". This involvement with abbreviations in the manuscript goes so far that one finds even "probl." for "problem" and "symb." for "symbol". Because of their number, and some particular problems they produced occasionally, taking care of the abbreviations made our editing task considerably harder, but this number tells that they cannot be neglected if one wants to leave a more precise record of Gödel's style (see the end of this introduction).

In the source version one may also find all the parts of the text crossed

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accords always with his, and we have not followed his editorial interventions.

out in the manuscript, with the indication that they were found crossed out, either by being really crossed out in the source version, or if they are too long, the crossing out is mentioned as an editorial comment within ⌊ and ⌋. We use ⌊ and ⌋ in the source version in connection with the abbreviations as we said above, and in general for other editorial comments too. (For example, we will have ⌊unreadable text⌋.)

In a few cases we have estimated that a crossed out part of the text is worth reproducing even in the edited version. (Gödel's crossing out a text need not mean dissatisfaction with it, but it may mean perhaps lack of time to use it in the lectures.) In one place it may compensate a little bit for a lost part of the text (see the footnote on p. **7**. of Notebook IV), in another (see the footnote on pp. **114**.-**115**. of Notebook VI), it completes what is needed for establishing that binary relations with composition and the identity relation make a monoid. (Composition of relations is called by Gödel "relative product", and his examples for it are with relations between *relatives*, nephew, son, brother, sister, uncle, father, grandfather, grandchild, child,. . . , which is etymologically inspirative.) A third such place, which is tied to Russell's understanding of definite descriptions (see pp. **123**.-**125**. of Notebook VI), is philosophically important.

Let us dwell for a moment at this third place, to justify our choice of reproducing the crossed out text. Gödel's says there that taking "The present king of France is bald" as meaningless is undesirable because whether the present king of France exists is an empirical question. He then continues: "Therefore it would depend on an empirical fact whether or not this sequence of words is a meaningful statement or nonsense, whereas one should expect that it can depend only on the grammar of the language concerned whether something makes sense." So Gödel asserts the primacy and independence of the understanding of language over empirical, i.e. epistemological, matters. The primacy of the linguistic over the epistemological (and presumably other philosophical concerns, like the ontological, or axiological) should be one of the main, if not the main, mark of the linguistic turn in twentieth century philosophy. Gödel's single sentence quoted above is more significant and more explanatory than thousands and thousands of others in the sea of ink spilled over the king's baldness.

The notes are written by hand in English in eight notebooks bound by a spiral, with however some loose leafs (four leaves on a different paper, not torn out from the notebook, without holes for the spiral, at the end of Notebook III with pp. **new page x-xiii**, nine torn out leafs towards the end of Notebook V including pp. **73.1-73.7**, and nine torn out leafs at the end of Notebook VII with pp. **new page iii-iv** and **1.-7.**). Gödel writes usually on the left pages, the back sides of the leafs, and he uses the right pages, the front side of the leafs, most often for inserted additions, or simply continuations of the text from the left pages. As insertion signs, one finds most often $\forall$ (which is not used in the manuscript for the universal quantifier), but also $\times$, and a few others. Insertions tied to these signs, as well as other insertions, often tied to $\underbrace{\qquad}$, but not continuations on the right pages, are marked in the source version with \ at the beginning of the insertion and / at its end. Sometimes one finds remarks and examples not possible to insert simply in the main text, and they are not to be found in the edited version. Since usually only the left pages are numbered, and the right page is usually associated with the left, we do not speak of left and right pages, but say, for example, that something occurs on the right of a certain page, or use similar forms of speaking.

There are no footnotes in the source version, because Gödel does not have them. (We do not interpret his insertions as footnotes.) All the editorial comments there are within $\lfloor$ and $\rfloor$. All the footnotes in the edited text are ours, and they are made of editorial comments.

In general we have strived to stay as close to Gödel's text as possible, at the cost of failing to follow standard usage. Gödel's manners in writing are sometimes strange, according to the contemporary standards, but they always make sense. (On pp. **47.-49.** of Notebook II he says, for example, "then and only then" for "if and only if", which one finds later. Instead of three dots as a punctuation mark he uses two—perhaps because he wants to abbreviate—but we have rendered that both in the source and the edited version in the usual triple way.)

We have corrected Gödel's not very numerous spelling mistakes, and did not keep in the edited text peculiar or foreign spelling (like "tautologie" and "geometrie"). If however an unusual spelling (like, for example, *caracter* instead of *character*) is permitted by the Oxford English Dictionary, then we kept it. We have not corrected Gödel's style in the notes, and we are aware that it is often on the edge of the grammatically correct, and perhaps even sometimes on the other side of the edge. In cases of doubt we opted for keeping his words. We made this choice because thereby the reader should be able to hear better Gödel lecturing, to hear his voice and not the voice of somebody else. Gödel had at that time no doubt his own foreign accent, which, since we ourselves are not native speakers of English, we did not want to replace with ours.

Gödel omitted in the notes many punctuation marks, in particular commas and quotation marks, but also full stops, presumably for the sake of abbreviating. We have added them, in the source version with $\lfloor$ and $\rfloor$ and in the edited text, together with some colons, only when we considered they are absolutely indispensable, but we did not want to add all of them that would usually be written. For example, Gödel practically never wrote commas before "then", and we did not add those.

Gödel was very sparing in using quotation marks. (Initial quotation marks he wrote in the German way ,, and not ".) He did not use them systematically for naming words and sentences. We did put them at many places where we were afraid understanding would be endangered, but at the cost of looking unsystematic, as Gödel, we did not restore them everywhere. We felt that in doing that, analogously to what we said in the preceding paragraph, we would be too intrusive, and get too estranged from Gödel's customs and intentions. Perhaps he did not omit quotation marks just for the sake of abbreviating, but wanted to use words autonymously, which might be related to his involvement with self-reference (see the end of this introduction). Once one becomes accustomed to this autonymous use, it hardly leads to confusion.

To make easier comparison with the scanned manuscript (which is the only one we have seen), we have standardized only slightly the numbers of the pages Gödel assigned to them there. These numbers are rendered in both the source and edited version with boldface Arabic figures, followed by a full stop, which is to be found in the manuscript, but not always, and also further figures, Arabic, Roman, or letters found in the manuscript; examples will come in a moment. We found five successive, not very systematic, numberings of pages in the manuscript starting from pages numbered **1**. in various notebooks. Some pages were left unnumbered by the numberings, and we introduced our own way of naming them, usually with the label **new page**.

We believe the first numbering is made of pp. **1**.-**26**. **I** of Notebook I (where a break occurs in that notebook). We will explain below why we think these pages of Notebook I should precede Notebook 0.

The second numbering starts with pp. **1-38**. of Notebook 0 (i.e. the whole of that notebook), followed by pp. **38.1 II**-**44**. **II** of Notebook I, followed by pp. **33**.-**55**.**2** of Notebook II, followed by pp. **56**.-**60**. of Notebook I, followed finally by pp. **61**.-**76**. of Notebook II. Our reasons for this complicated arrangement are in the sense of the text. For example, the involvement of

Notebook II in this numbering has to do with the presentation of the axiom system for propositional logic (see Section 1.1.9 in the edited text below). We must warn however that though in this numbering the page numbers from different notebooks sometimes fit perfectly, and follow the sense, sometimes the fitting is somewhat less than perfect.

We have rearranged the page order in our edited version as the first and second numberings require. In the source version the original order from the scanned manuscript is kept in general, and also for the pages involved in these numberings. The order of pages required by the remaining three numberings are the same in the edited and source version and in the scanned manuscript, with a small exception which we will mention in a moment.

The third numbering is from the initial, first, p. **1**. of Notebook III up to p. **53**. of that notebook.

The fourth, longest, numbering is from the second p. **1**. of Notebook III, which is close to the end of the notebook, up to p. **157**. of Notebook VII, following more or less regularly the order of the notebooks and the numbering in them.

A small rearrangement guided by subject matter is made in the edited version in the last part of Section 1.1.10, where guided by subject matter four pages from Notebook IV not numbered in the manuscript have been inserted, which has made possible a perfect fitting in Section 1.1.14 *Sequents and natural deduction system*.

The fifth, last and shortest, numbering is made of pp. **1**.-**7**. of Notebook VII, at the very end.

Zero precedes one, and presumably because of that, in the scanned manuscript Notebook 0 precedes Notebook I, while in §1.II of [Dawson 2005] one finds that Notebook I "appears to be a rewritten, somewhat condensed version" of Notebook 0. It is however not clear in relevant cases that condensation from 0 to I is made, and sometimes the opposite, addition, from I to 0 seems to be at work. Sometimes even the text in Notebook 0 is shorter than the corresponding text in Notebook I, from which it seems to have been obtained by tidying up (cf. in the source version the text pp. **20**.-**21**. of Notebook 0 with the approximately twice longer corresponding text on pp. **15**.-**16**. of Notebook I). We want to present now additional reasons for believing that Notebook I precedes Notebook 0, and that Notebook 0 together with the parts mentioned in the second numbering above is written later and may be considered to supersede the pages of Notebook I in the first numbering.

From p. **4**. until the end of p. **21**. of Notebook I propositional variables are written first mostly as capital P, Q and R, which are later on alternated with the lower-case p, q and r. In the edited version they are all written uniformly as lower-case, because when they alternate they might be confusing, while in the source version they are as in the manuscript. After p. **21**. of Notebook I and in Notebook 0 the lower-case letters only are used for propositional variables. This usage is kept in Notebook II and later, and the capital letters starting from p. **58**. of Notebook I, which belongs to our second numbering, and later, are used as schematic letters for formulae. The notation in Notebook 0 seems to be a correction of that in Notebook I.

Before p. **42**. **II** of Notebook I the signs + and −, which were used in the notes for the 1935 Vienna course, are used instead of T and F for naming truth values. The letters T and F are to be found in Notebook 0, on pages of Notebook I that belong to our second numbering, and they are used regularly in Notebook II and later. In the edited text we did not try to replace + and − by T and F, because no confusion is likely.

The pages numbered in the manuscript with the suffix I in Notebook I, which belong to our first numbering, could be superseded by pages after p. **23**. of Notebook 0, which leave a better impression and belong to our second numbering. The suffix II added in the manuscript to some later pages in Notebook I would indicate that these pages belong to the second numbering.

In Notebook I decidability is considered with tautologies on pages that make Section 1.1.7 *Decidability for propositional logic* of the edited text. In Notebook 0 decidability is not considered, but it is considered more thoroughly on pp. **41**. **II-44**. **II** of Notebook I, which belong to our second numbering.

The axioms of the system for propositional logic would appear for the first time on p. **53**. of Notebook II, which until the end Notebook II is followed by a preliminary discussion of the role of primitive rules of inference in logic (we consider this matter below in a more philosophical spirit), but no such rule is given. The primitive inference rules are to be found on pp. **56**.-**59**. of Notebook I, and after them the four axioms are given again on p. **60**. of Notebook I. This induced part of the order in our second numbering.

On pp. **11**.-**12**. of Notebook I Gödel writes something like handwritten o, which we put (or perhaps σ), for exclusive disjunction, while on pp. **16**. and **18**. of Notebook 0 he has for it $\circ$, which is then again to be found on p. **44**. of Notebook II.

On the same pages pp. **11**.-**12**. of Notebook I, and also on p. **7**. of the same notebook, one finds a number of times a crossed out word "wrong" replaced by "false". In Notebook 0 "wrong" is not to be found and "false" is used regularly, while later "wrong" occurs here and there, but "false" predominates.

At the very beginning of the notes, the programme of the course is stated together with a reprobation of traditional logic (which we will consider below in this introduction). Citing the source version, a sentence in that part starts with: "What the textbooks ~~give~~ and also what Arist.⌊otle⌋ gives is a more or less arbitrary selection of the \ infinity of / ⌊the⌋ laws of logic" on p. **1**. of Notebook I, and with: "What the trad⌊itional⌋ logic gives is a more or less arbitrary selection from the infinity of the laws of logic" on pp. **1**-**2**. of Notebook 0. We have not gone over the matter systematically, but it seems to us that this is an indicative sample of what happens when one passes from Notebook I to Notebook 0. In Notebook I we have "selection of the infinity of laws of logic", where "infinity of" has been inserted ("⌊the⌋" means that the article has been added by us in the edited version), while in Notebook 0 we have "selection from the infinity of the laws of logic", which is less ambiguous and better English. Note, by the way, that Aristotle and textbooks are not mentioned here in Notebook 0 (on p. **1**. of Notebook 0 a mention of textbooks a little bit earlier has been crossed out, as marked by a footnote in the edited version).

We conclude our discussion about Notebook I preceding Notebook 0 with a detail that sets Notebook 0 apart, and that together with the number of that notebook may point in the other direction. On the front cover of Notebook 0 one finds "Vorl. Log.", while on the front covers of all the remaining notebooks one finds "Log. Vorl.", except for Notebook VII, where "Logik Vorl." is written (see the source version).

Gödel's text has neither chapters nor sections, nor an explicit division into lectures. The edited version and the source version make two chapters in this book. We have divided the edited version into two parts, the first about propositional and the second about predicate logic, and we have further divided these parts into sections which, as the parts, we have named with our own words. Our titles of the parts and sections are not mentioned in the source version. For them we use standard modern terminology and not Gödel's. We put "connectives" instead of "connections". Gödel did not use the expressions "functional completeness", "disjunctive normal form", "conjunctive normal form", "sequents", "natural deduction", "first-order languages", "valid formulas" (he uses "tautology" also for these formulae, or he says that they are universally true). He uses the term “class” rather than “set”, and we have kept it for naming Sections 1.2.7 and 1.2.8 in the edited text. Our table of contents below is not exactly the same as that given in [A. & D. 2016]. The present one is more detailed and follows more closely the manuscript, including repetitions in it. We have added moreover to the edited text an index for it.

Gödel did not pay very much attention in the notes to the division of the text into paragraphs, and where we found it very desirable, following either the sense of the text or rather the excessive length of the paragraphs in the manuscript, we introduced new paragraphs, with due notice, using ⌊new paragraph⌋, in the source version. We did not introduce them however at all places where this might have been done, following a policy similar to the one we had with punctuation marks.

Some, but not much, of Gödel’s text is unreadable and a very small part of it is in shorthand. Sometimes it is not clear whether one has to do with shorthand or unreadable text. We have not tried to decipher the shorthand in the source version, because practically everywhere it occurs in parts omitted in the edited version, which do not belong at all to the main text, and sometimes are not directly about logic (as, for example, in the theological remarks at the beginning of Notebook VII). We did not find we need this decipherment. The unreadable portions of the text are marked with the words “unreadable text”, “unreadable symbol”, or something related.

Pages written not very systematically, not numbered, with lists of formulae, jottings, and some unreadable text, crossed out to a great extent, have been rendered as far as possible in the source version but not in the edited one. We did not want to be too intrusive by making a selection in this text, which we estimate should not all belong to the edited version. There are thirteen such pages at the end of Notebook III. Notebook VII starts with nine, not numbered, pages of remarks and questions mostly theological, partly unreadable, partly in shorthand, and all seemingly not closely related to the remaining notes for the course. They are rendered as far as possible in the source version but not in the edited one. The text crossed out in the manuscript is not in the edited version.

The underlined parts of the manuscript have in principle been rendered in the edited version by italics. The underlining has however been kept in derivations where it can play a special role.

As we said in [A. & D. 2016] (see the section *Major problems and branches of logic*), [Hilbert & Ackermann 1928] influenced Gödel in general, and that influence is to be found in the Notre Dame course too. (This influence might be seen in details like the remarks on the Latin *aut* and *vel* on p. **9**. of Notebook 0, which follow [Hilbert & Ackermann 1928], Section I.§1, but Gödel also mentions *sive*... *sive* on p. **7**. of Notebook I.) In the notes Gödel does not use the expressions "formal language" and "inductive definition", and does not have a proper inductive definition of the formal language, i.e. of the formulae, of propositional logic (he comes nearest to that on pp. **11**. and **15**. of Notebook 0 and p. **8**. of Notebook I). The formal language of propositional logic is not defined more precisely in [Hilbert & Ackermann 1928], though a formal language of first-order predicate logic is defined by a regular inductive definition in Section III.§4. In the Notre Dame notes however, the formulae of predicate logic are not defined more precisely than those of propositional logic (see pp. **32**.ff of Notebook IV). It seems that in many textbooks of logic, at that time and later, and even today, clear inductive definitions of formal languages might be lacking, the matter being taken for granted.

In the precise inductive definition of formulae in [Gödel 1931] (Section 2, pp. 52-53 in the *Collected Works*), his most famous paper, Gödel has the clauses that if a is a formula, then $\sim(a)$ is a formula, and that if a and b are formulae, then $(a) \vee (b)$ is a formula. This definition excludes outermost parentheses, but in complex formulae it puts parentheses around propositional letters and negations, where they might be deemed unnecessary. This way of dealing with parentheses should explain why on pp. **14**.-**15**. of Notebook 0 (and occasionally also elsewhere, as on pp. **23**.ff of Notebook III) it is taken that there are parentheses around negations, as in $(\sim p)$, which are not customary, and that there should be a convention that permits to omit them.

To prefix the universal and existential quantifiers (x) and $(\exists x)$ square brackets are put in the notes around formulae before which they are prefixed, which is also neither customary nor necessary, as noted on p. **41**. of Notebook IV, where in some cases it is permitted, but not required, to omit these brackets. As in some other matters of logical notation, neither the convention to write the brackets nor the permission to omit them are followed systematically (see pp. **32.a**ff of Notebook IV). We have not tried to mend always this and similar matters in the edited text. Besides corrections of slips of the pen, found in formulae as well as in English, but not very numerous, we have made changes of what is in the manuscript in cases where we estimated that understanding would be hampered.

Gödel's usage in the notes is not very systematic and consistent, neither concerning formalities of logical notation, nor concerning matters of ordinary English, including punctuation marks (which he does not use as much as it is usual). One should however always bear in mind that the notes were presumably meant only for himself, and he could correct in the lectures whatever irregularity they contain. This matter concerns also sometimes the meaning of his text, which taken literally is not correct. He speaks, for instance, nearly always of substitution of *objects* and not of their *names* for individual variables (on p. **42**. of Notebook IV one finds, for example, "the free variables are replaced by individual objects"). On p. **138**. of Notebook VI he says "for any arbitrary object which you substitute for x", but three lines below he says "if you substitute for x the name of an arbitrary object". On p. **139**. of Notebook VII he has "if you substitute for x the name of an arbitrary object", with "the name of" inserted later (which in our source version is rendered with \ and /). So one may take that Gödel had always in mind the correct statements mentioning names, which at most places he omitted for the sake of abbreviating, which he relied on very much. (It is also possible that sometimes, except where names are mentioned, by *substituting* an object for a variable Gödel meant *interpreting* the variable by the object.)

Gödel's definition of tautology for propositional logic (see pp. **33**. of Notebook 0 and **25. I**. of Notebook I) and valid formula, i.e. tautology or universally true formula in his terminology, for predicate logic (see p. **45**. of Notebook IV) are not very formal. His definitions could be taken as defining syntactical notions based on substitution, if this substitution is not understood as model-theoretical interpretation (cf. the parenthetical remark at the end of the paragraph before the preceding one). The word "model" does not however occur in the notes, and the notion, which is somehow taken for granted, is not introduced with much detail.

Concerning tautologies of predicate logic, one finds on p. **54**. of Notebook IV and p. **55**. of Notebook V: "An expression is a tautology if it is true in a world with infinitely many individuals, i.e. one can prove that whenever an expression is universally true in a world with infinitely many objects it is true in any world no matter how many individuals there may be and of course also vice versa." Gödel says that he cannot enter into the proof of that. (For this matter one may consult Section III.§12 of [Hilbert & Ackermann 1928].)

Gödel seems parsimonious by relying a lot on abbreviations, but he does not spare his energy and time in explaining quite simple matters in great detail, and in repeating himself. He addresses beginners, and does not forget that they are that. This might be a reason to add to those mentioned in the following concluding remark in §1.II of [Dawson 2005] concerning the Notre Dame notes: "Although the material is standard, the choice and ordering of topics, as well as some of the examples that are discussed, may well be of pedagogical interest." In the remainder of this introduction, we will give reasons that should be added to those given in [A. & D. 2016], [D. & A. 2016] and [D.&A. 2016a] to justify our belief that the interest of these notes is not just pedagogical.

Our involvement with Gödel's notes from Notre Dame started with an interest in Gödel's views concerning deduction, about which we wrote in [D. & A. 2016] and [D.&A. 2016a]. This was the main reason for our getting into the project, which, as can be gathered from [A. & D. 2016], led to other matters concerning the course that we found interesting. (Also, one of us taught a logic course as a visiting professor at Notre Dame when he turned 33.) Concerning deduction, we would like to add here that on pp. **69**.-**70**. of Notebook II Gödel commends derived rules and says "in our system we cannot only derive formulas but also new rules of inference". We believe this short remark is in accordance with our discussion in [D.&A. 2016a] and [D. & A. 2016] of Gödel's natural deduction system of Notebook IV and his recommendation of it in Notebook III. Gödel's remarks about rules of inference on pp. **52**.-**55**.**2** at the end of Notebook II, which in the edited text are at the beginning of Section 1.1.9 *Axiom system for propositional logic*, are relevant too for Gödel's opinions about deduction. Gödel says there that if rules are not formulated explicitly and derivability is understood as, for example, in geometry, where it means "follows by logical inference", then "every logical law would be derivable from any other" (p. **55**.**1** of Notebook II; cf. the second p. **4**. towards the end of Notebook III).

In the edited text we entitled Section 1.1.4 of Notebook 0 and the corresponding Section 1.1.1 of Notebook I *Failure of traditional logic—the two gaps*. Before dealing with the two gaps, let us survey other aspects of this failure in connection with matters in the notes. There is first the arbitrariness and narrowness of the selection of the type of logical form to be investigated. The logical words selected are not completely pure (quantifiers are meshed with the connectives in the Aristotelian a, e, i, o forms), and they do not cover completely the propositional connectives, as Gödel points out towards the end of Section 1.2.8 of the edited text (this is a matter in the sphere of functional completeness, treated by Gödel in Section 1.1.8 of the edited text).

These words are also incomplete because they do not cover the quantifiers, as it is clearly shown by the envisaged axiomatization of Aristotelian syllogistic as a formal theory of propositional logic in Section 1.2.8 *Classes and Aristotelian moods* of the edited text. (We have said in § 16 of [A. & D. 2016] that Łukasiewicz was working on such a presentation of Aristotelian syllogistic not much later than Gödel in the Notre Dame course, if not at the same time, and they approached the subject in very much the same manner. This was a short while before the invasion of Poland and the outbreak of the Second World War, when Gödel was back in Vienna.)

Relations of arity greater than the arity one, which properties have, are also left out in the Aristotelian approach, and this is another crucial incompleteness, as Gödel says in the third paragraph of Section 1.2.1 *First-order languages and valid formulas* of the edited text, because these relations are more important than properties "for the applications of logic in mathematics and other sciences". He also notes in the following paragraph of that section: "Most of the predicates of everyday language are relations and not properties."

Traditional logic deals exclusively with unary predicates, tied to properties, but it is incomplete also because it does not take all of them into account. Those which have an empty extension are left out, and this is detrimental for the use of logic, as Gödel says in Section 1.2.6 *Existential presuppositions* of the edited text. First, logic becomes dependent on empirical matters, and it also becomes impossible to use logic for answering in mathematics or elsewhere the question whether there is something that satisfies a property. Like leaving out zero in mathematics, it makes also the theory unnecessarily more complicated and uglier, if it does not end up in confusion and outright mistakes with the four wrong moods among the 19 moods, or with the conviction that no conclusion can be drawn where this is not the case (see the end of Section 1.2.8).

In Sections 1.1.4 and 1.1.1 Gödel speaks about traditional logic failing to present logical laws as theorems of a deductive system. Occasionally in the past one heard boasts concerning this matter, which were based neither on a proof nor even a clear conception of the completeness in question. With a slight knowledge concerning classes and a few operations on them, which is based on a small, simple and intuitive fragment of propositional logic, of which Aristotelian logic is not aware, all the correct 15 Aristotelian moods are contained in a single formula (see Section 1.2.8). Decidability, which Gödel calls completeness (see the remarks about the first gap below), is beyond the narrow horizon of traditional logic.

So taking into account several kinds of completeness, traditional logic failed to reach any of them. It is a complete failure. Traditional logic seems at first glance to be much present in Gödel's course, but only in the Stoic's anticipatory discovery of connectives and propositional logical form there is something mentioned with approval—in the Aristotelian heritage nothing.

This complete failure of traditional logic in matters of completeness should certainly be taken into account in the explanation of the waste of the realm of traditional logic, which Greek mathematicians and most of the later ones ignored in their work, while some, like Descartes, condemned severely, centuries ago. Gödel's measured but thorough condemnation is made in the light of various aspects of completeness, a modern theme developed by him with success in logic and mathematics.

Gödel says that his chief aim in the propositional part of the course is to fill two gaps, solve two problems, which traditional logic failed to deal with, let alone solve (see the bottom of p. **3**. of Notebook 0 and the bottom of p. **2**. and the top of p. **3**. of Notebook I). The first is he says the problem of completeness of logical inference and logically true propositions, which he explicates as decidability, and the second is the problem of showing how all of them can be deduced from a small—he says "minimum"—number of primitive laws. He considers the first problem solved by showing that the notion of tautology is decidable (see the bottom of p. **43**. II of Notebook I), and the second is solved by proving a deductive system for propositional logic complete (i.e. the sets of provable formulae and tautologies coincide; see the second p. **2**. towards the end of Notebook III). The two analogous problems for predicate logic are considered on p. **47**. of Notebook IV. Gödel mentions that the second completeness problem was solved positively, and he gives indications concerning the negative solution of the first completeness problem, i.e. decidability, without entering into the proofs. He mentions the decidability of the monadic fragment.

For propositional logic Gödel considers (at the end of p. **43**. II of Notebook I) that providing a decision procedure is even more than what is required for solving the first problem, as if he thought that providing concretely such a procedure (which is moreover easy to understand) is more than showing decidability nonconstructively. Usually today, completeness is understood in such a way that showing just the recursive enumerability of the set of tautologies is enough for it, and showing the recursiveness of that set is not compulsory. Decidability, i.e. the recursiveness of the set of tautologies, amounts to showing that both this set and its complement with respect to the set of formulae, are recursively enumerable, and so it makes sense to call decidability too *completeness*; it is completeness in a stronger sense. Gödel in any case distinguished the first problem, and the completeness involved in this problem, from the second problem of showing completeness with respect to a deductive system. From a positive solution of the first problem one can deduce the recursive enuberability of the set of logical laws, but that is not enough for the second problem, which awaits to be solved. By not reducing proof theory to recursion theory, Gödel took deduction as a separate important matter.

In that context, speaking of rules of inference Gödel says: "And of course we shall try to work with as few as possible." (p. **54**. of Notebook II) The "of course" in this sentence reflects something still in the air at the time the course was given, about which we spoke in Section 5 of [D.&A. 2016a]. Gödel's advocacy of minimality is also related to the problem of independence of the axioms, with which he dealt in Section 1.1.12 of the edited text concerning his axiom system for propositional logic. This is besides completeness and decidability one of the main problems of logic, to which many investigations in set theory, in which Gödel was involved too, were devoted. We believe that his advocacy of minimality has however also to do with the following.

We said above several times that Gödel used abbreviations very much. The economy brought by them is not only, so to speak, physical—with them less paper is needed, less ink, the reading is quicker. This economy is also of a conceptual kind. The Chinese way of writing need not have evolved from abbreviations, but it is as if it did. By moving away from the phonetic way of writing we do not represent concepts indirectly through the mediation of spoken words, which are represented in our writing. We represent the concepts directly. The written word "two" represents the number two indirectly through the mediation of the spoken word, while the figure 2 represents it directly. The written word "prop." moves away from the representation of the spoken word "proposition" (and the context is practically always sufficient not to confuse it with the "prop." of "property"). The abbreviation "log." in our example above stands for different words of different grammatical categories, as a Chinese character does. The Chinese way and the similar mathematical one are eminently reasonable, and bring advantages once one becomes accustomed to them.

Mathematical notation is far from phonetic. If something phonetic is still present in it, it is through abbreviations, or traces of abbreviations, often initial letters, as with functions being usually called f. There might be something mathematical in Gödel's inclination towards abbreviations.

Gödel's lectures end in the notes with Section 1.2.10 *Type theory and paradoxes* of the edited text (pp. **127**.-**140**. of Notebook VI and **137**.-**157**. of Notebook VII, which precedes Section 1.2.11 *Examples and samples of previous subjects*, which does not seem to be a lecture), where he presents Russell's paradox not explicitly as a set-theoretical matter, but through the predicate Φ, read "impredicable", such that $\Phi(x)$ is equivalent with $\sim x(x)$ (see p. **142**. of Notebook VII; he follows there [Hilbert & Ackermann 1928], Section IV.§4). Then on pp. **149**.-**156**. of Notebook VII he argues forcibly that self-reference (his term is "self-reflexivity") should not be blamed for the contradiction. He says that rejecting self-reference, which inspired Russell's theory of types, both in its ramified and in its simplified form, excludes many legitimate arguments based on self-reference, which do not lead to contradiction and are necessary for building set theory (pp. **155**.-**156**. of Notebook VII). The contradiction in the paradoxes is due to the illegitimacy of taking that there is a complete, achieved, totality of all objects—or to put it in other words, the impossibility to achieve completeness in the extensional realm.

It would be in Gödel's style to write: "Abbr. is an abbr". The turn towards the conceptual here need not however be simply mathematical, because the self-reference involved could be akin not only to that made famous by [Gödel 1931] but also to the intensional logic of the future (about which we said something in Section 5 of [D. & A. 2016]), where with legitimate self-reference the achievement of completeness is expected.

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The page numbers below within square brackets, for example, [0: 1.] and [V: 73.1], are to be read as "p. **1**. of Notebook 0" and "p. **73.1** of Notebook V" respectively. They come from the page numbers in the manuscript and the source version, which are rendered in boldface in the text below with the brackets ⟦ ⟧. These page numbers refer to the first page of the section in question in the source version.

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An edited version is given of the text of Gödel's unpublished manuscript of the notes for a course in basic logic he delivered at the University of Notre Dame in 1939. Gödel's notes deal with what is today considered as important logical problems par excellence, completeness, decidability, independence of axioms, and with natural deduction too, which was all still a novelty at the time the course was delivered. Full of regards towards beginners, the notes are not excessively formalistic. Gödel presumably intended them just for himself, and they are full of abbreviations. This together with some other matters (like two versions of the same topic, and guessing the right order of the pages) required additional effort to obtain a readable edited version. Because of the quality of the material provided by Gödel, including also important philosophical points, this effort should however be worthwhile. The edited version of the text is accompanied by another version, called the source version, which is quite close to Gödel's manuscript. It is meant to be a record of the editorial interventions involved in producing the edited version (in particular, how the abbreviations were disabridged), and a justification of that later version.

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